

How the Pyramid Builders May Have Found Their True North

by Glen Dash

In 2011–2012 AERA began a comprehensive resurvey of the base of the Great Pyramid, the Sphinx, and some of the other great monuments of Giza. We excavated our own archives as well, and with that data AERA associate Glen Dash attempted to answer some of the fundamental questions that scholars have puzzled over for more than a century: What are the exact dimensions of the base of the Great Pyramid? How well is it oriented to true north?* In this issue, he addresses another question scholars have debated: how did the ancient Egyptians manage to orient their pyramids so accurately to true north?

The builders of the Great Pyramid of Khufu aligned the huge monument to true north to within six minutes of arc, or one tenth of a degree.¹ Scholars have described that achievement as extraordinary, astonishing, and brilliant. How they managed to do that has long been debated. In this article we will examine four prominent theories and test one.

Old Kingdom texts, pictorial representations, and surviving tools offer us few clues as to how the Egyptians built their pyramids, let alone how they aligned them. In his book *The Pyramids of Egypt* (1947/1993), I. E. S. Edwards summed up the situation: “Extant Egyptian records, whether written or pictorial, throw no light on the methods employed by the builders of the pyramids either in the planning or in constructing their monumental works.”² Scholars, therefore, have had to formulate theories without much help from the Egyptians themselves.

The Imperishable Ones

Why did the Egyptians need to align their pyramids with cardinal points? The answer may lie in their vision of the Netherworld. The *Pyramid Texts*, first inscribed on the walls

*Glen Dash, “New Angles on the Great Pyramid,” *AERAGRAM* 13-2, Fall 2012, pages 10–19. All back issues of *AERAGRAM* are available for free download at our website: aeraweb.org.

of the burial chamber of 5th Dynasty king Unas (c. 2356–2323 BC), describe eternity and the deceased’s connection to the celestial world. The king was to “go forth to the sky among the Imperishable Ones” and “go around the sky like the sun.”³ The “Imperishable Ones” were the circumpolar stars, so named because they never set below the horizon.⁴ The king’s spirit may have been guided on its journey by the orientation of the pyramid’s inner spaces.⁴ At their northern end, the corridors of many pyramids lead to a “descending passageway” angled up at the circumpolar stars “like a telescope.”³

While the *Pyramid Texts* may help us understand why the Egyptians aligned their pyramids to cardinal points, we do not know why they needed to do it with such precision. As Mark Lehner has said, “For the royal designers, such exactitude may have been imbued with symbolic and cultic significance that now eludes us.”³

The Vault of the Heavens

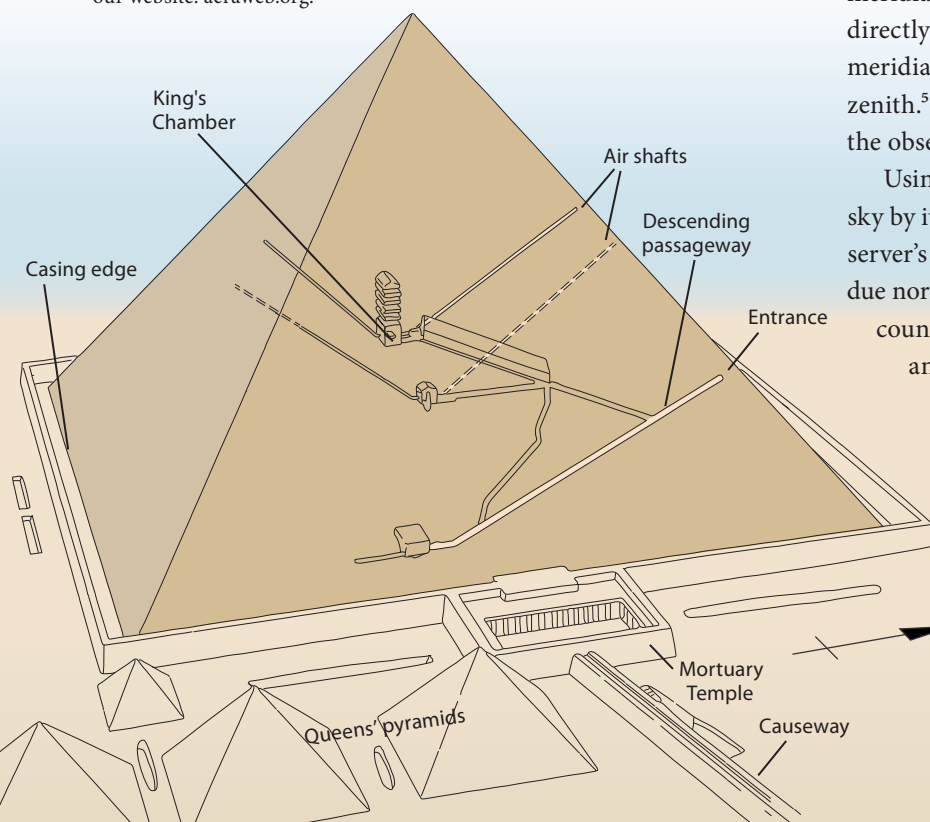
To astronomers, due north and south are said to lie on the meridian line, a line that connects the geographic South Pole to the North Pole (facing page, top left). An observer standing anywhere on the earth’s surface looking north finds the meridian line running between his or her feet. The location directly above the observer’s head is known as the zenith. The meridian circle arcs overhead connecting north, south, and the zenith.⁵ The meridian circle lies in a plane perpendicular to the observer’s horizon.

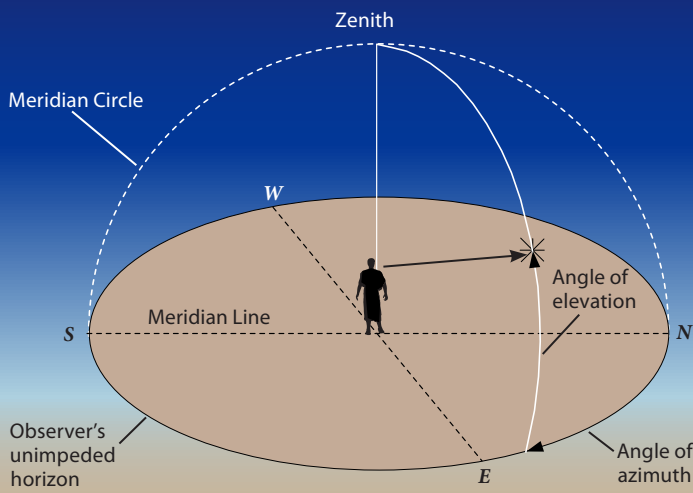
Using these lines and circles, we can locate any star in the sky by its elevation—the vertical angle of the star above the observer’s unimpeded horizon—and its azimuth—its angle from due north along the horizon. A clockwise angle is positive, and counterclockwise, negative. We measure angle in degrees and minutes of arc, with 60 minutes in one degree.

Edwards’s Circle

Edwards claimed that the Egyptians located the meridian with the aid of an artificial horizon—a

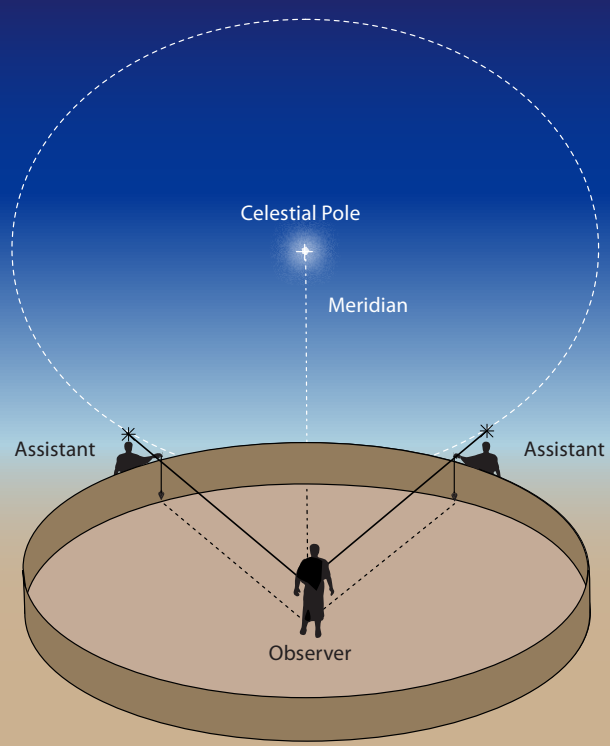
Inside the Great Pyramid. The body of the king was laid to rest in the King’s Chamber. Its spirit moved among its internal spaces and may have exited out via the descending passageway. Scholars have attempted to determine the pyramid’s intended alignment by examining the orientation of its casing and its passageways. (After *The Complete Pyramids* by Mark Lehner, London, Thames & Hudson, 1997.)





Above: Mapping the sky. The meridian line lies in the direction of due north. Any object in the sky can be located by its azimuth and elevation.

Right: Edwards's method. Edwards theorized an observer placed inside a circular wall that served as an artificial horizon. An assistant marked the location where a certain star rose and then, hours later, set. The observer bisects the angle formed in order to locate the meridian.



circular wall built around an observer, tall enough to exclude all but the night sky from view.² The observer stood at a fixed spot at the center of the circle and watched over a pole or through a sight as a particular star rose over the wall in the east. An assistant marked the location where the observer saw the star rise. Later the observer watched the same star as it set over the wall in the west. The assistant marked that spot as well. The locations of the two spots were extended to the ground through the use of a plumb bob. Bisecting the angle formed by the two locations and the observer's position derived the meridian.³

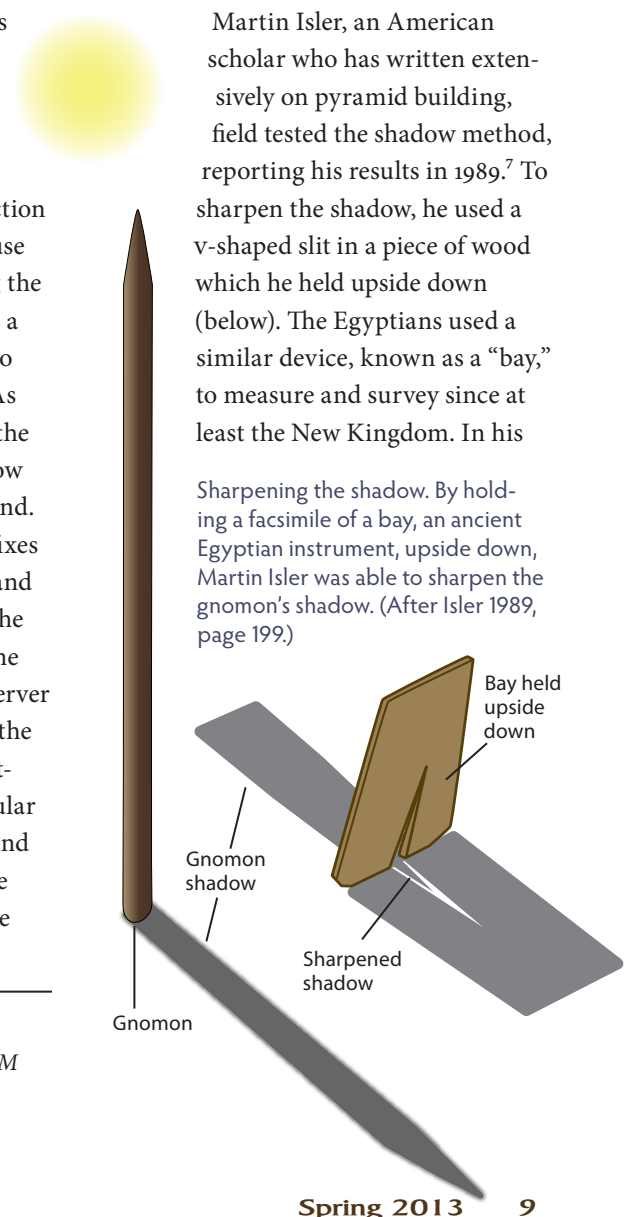
However, Edwards never field tested his theory. If he had, he may have found it difficult. To achieve an accuracy of 6 minutes of arc, he would have needed to establish the relative positions of the star's rising, setting, and the center of the circle accurately to better than 2 parts in 1000. If his circle were 3 meters in diameter, that translates to a total error of just 6 millimeters. Edwards acknowledges that in order to achieve that, the wall would have to have been almost perfectly round and level, a feat other

scholars have doubted the Egyptians could have achieved.⁶

Isler's Shadows

Perhaps the most ancient instrument we have for determining direction is the simple vertical pole. We can use a pole to find the meridian by using the "shadow method."* An observer sets a pole—a *gnomon*, Greek for "one who knows"—vertically in the ground. As the day passes, the observer marks the location of the tip of the sun's shadow as it moves in an arc along the ground. At the end of the day, the observer fixes a string to the base of the gnomon and draws a circular arc which crosses the shadow pattern at two points. If done over perfectly level ground, the observer will find that a line drawn through the intersecting points runs exactly east-west. The meridian runs perpendicular to this east-west line and can be found by bisecting the angle formed by the two intersections and the base of the gnomon.

* The shadow method is described in "North by Northwest: the Strange Case of Giza's Misalignments" by Glen Dash, *AERAGRAM* 13-1, Spring 2012, pages 10–15.



Martin Isler, an American scholar who has written extensively on pyramid building, field tested the shadow method, reporting his results in 1989.⁷ To sharpen the shadow, he used a v-shaped slit in a piece of wood which he held upside down (below). The Egyptians used a similar device, known as a "bay," to measure and survey since at least the New Kingdom. In his

Sharpening the shadow. By holding a facsimile of a bay, an ancient Egyptian instrument, upside down, Martin Isler was able to sharpen the gnomon's shadow. (After Isler 1989, page 199.)

tests, conducted in Wilton, Connecticut, on September 7, 1988, Isler achieved an accuracy of 19 minutes of arc using a gnomon just 60 centimeters high.

Petrie's Elongations

Flinders Petrie, whose seminal 1880–1882 survey of the Great Pyramid is still widely used today, believed the pyramid builders found due north by following the pole star. In his 1883 book, *The Pyramids and Temples of Gizeh*, he tersely described the method he thought they used:

The setting out of the orientation [of the Great Pyramid]... would not be so difficult. If a pile of masonry 50 feet high was built up with a vertical side from North to South, a plumb-line could be hung from its top, and observations could be made, to find the places on the ground from which the pole-star was seen to transit behind the line at the elongations, twelve hours apart. The mean of these positions would be due South of the plumb-line and about 100 feet distant from it; on this scale 15 [seconds] of angle would be about 1/10 inch, and therefore quite perceptible.⁸

A version of Petrie's method is illustrated below and on the facing page. A plumb line is suspended from Petrie's north-south masonry wall. An observer watches for the pole star to transit behind the plumb line from beyond a low bench which holds a moveable sight. The bench and sight, suggested by the Czech archaeologist Zbyněk Žába,⁹ make the observer's task easier.

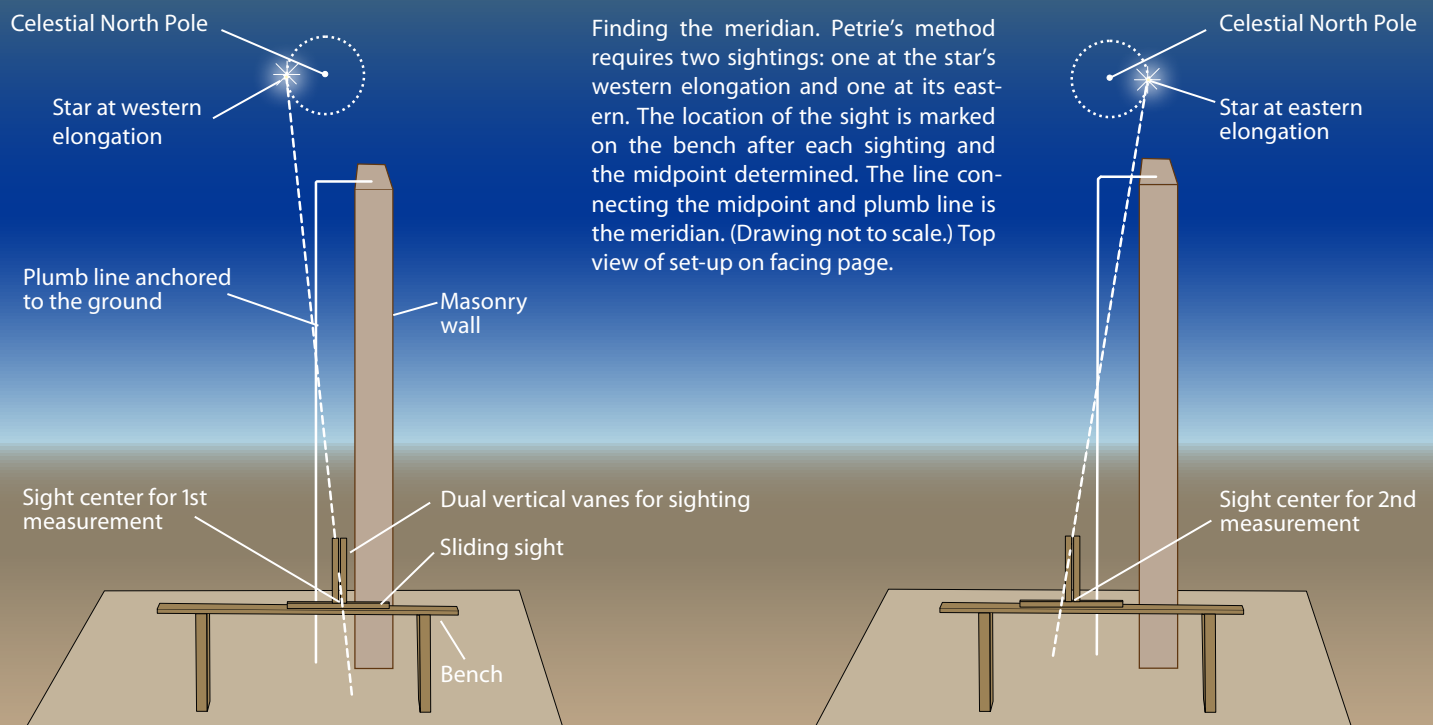
The purpose of the whole arrangement is to record the extreme movements of the pole star, as shown below. Like all other stars of the northern hemisphere, the pole star circulates

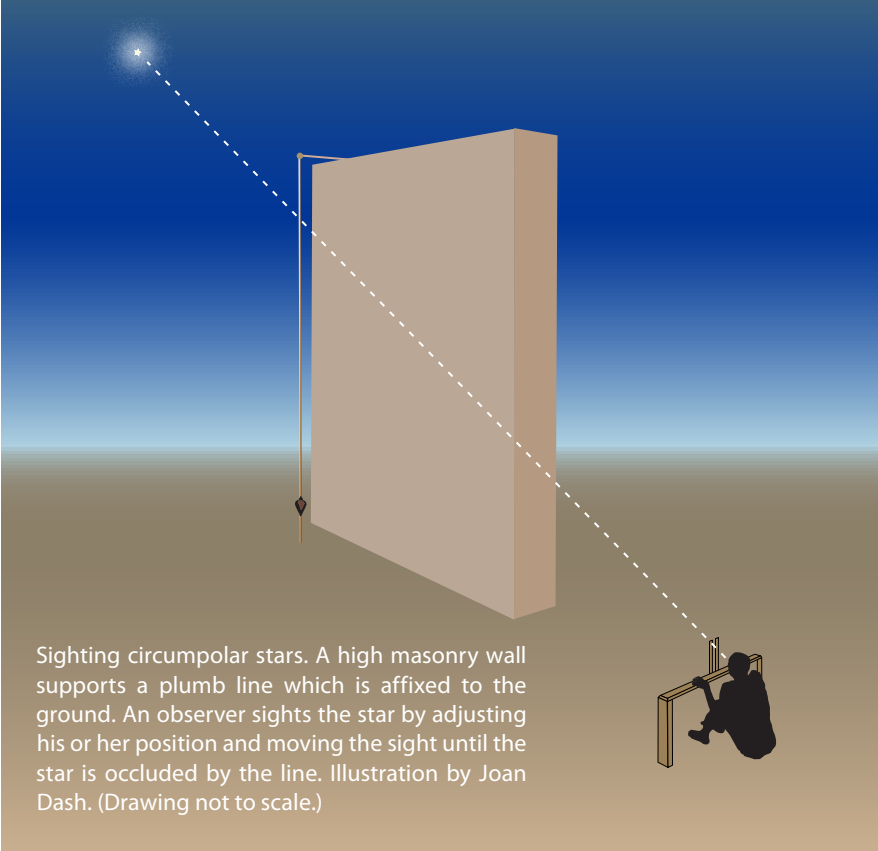
around the north celestial pole counterclockwise. Today, the pole star is Polaris, about one degree distant from the celestial pole. At the time the Great Pyramid was built, it was Thuban, about 2 degrees distant. As the pole star rotates around the north celestial pole, it passes sequentially through its highest point in the sky (upper culmination), its westernmost point (western elongation), its lowest point (lower culmination), and its easternmost point (eastern elongation).

Viewing the pole star through the sight's parallel vanes, the observer tracks the movement of the star by moving the sight along the bench from east to west or west to east until the pole star disappears behind the rope. The star eventually reaches one of its elongations, and when it does, the observer marks the location of the center of the sight on the bench. The observer continues to watch until the pole reaches its other elongation and then marks that location as well. The observer then makes a third mark on the bench precisely between those two. A line drawn between this third mark and the rope should lie on the meridian.

Testing Petrie's Theory

Petrie never field tested his theory. Therefore, in the fall of 2012, I did so at my home in Pomfret, Connecticut. Instead of a 50-foot masonry wall, I used a wooden support mounted on a second story porch which held a 3/16 inch-diameter rope about 10 meters long. The rope was secured near the ground and plumbed vertically using a total station, a survey instrument. The observer's bench stood approximately 8 meters south of the plumbed line and was aligned roughly east-west by eye. The sight was constructed from two carpenter squares and wood,





Sighting circumpolar stars. A high masonry wall supports a plumb line which is affixed to the ground. An observer sights the star by adjusting his or her position and moving the sight until the star is occluded by the line. Illustration by Joan Dash. (Drawing not to scale.)

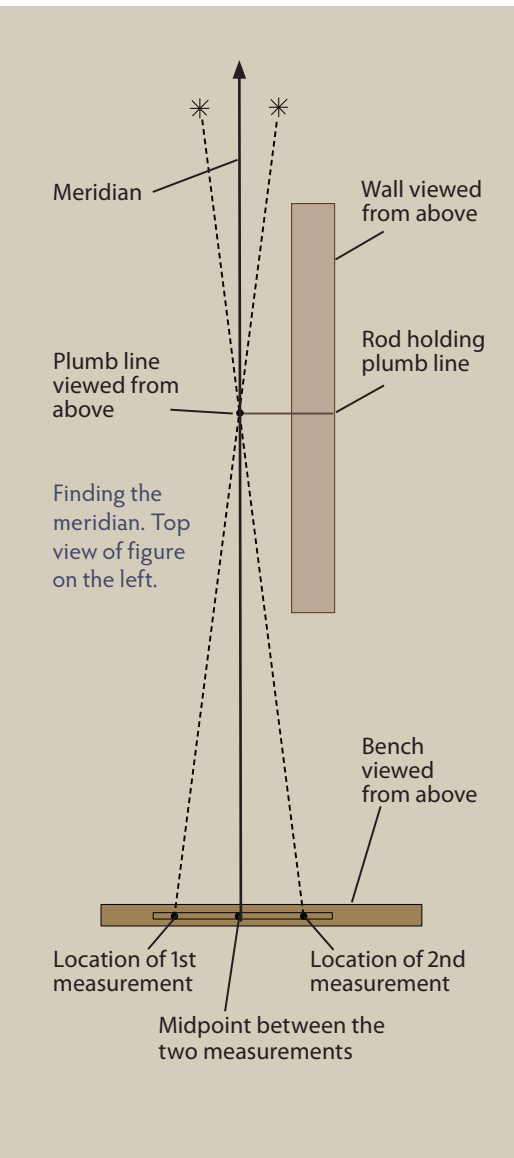
with a 1/2 inch separation set between the vertical legs of the carpenter squares for viewing.

The arrangement proved easy enough to use. I moved the sight from side to side and repositioned myself until Polaris disappeared behind the rope as viewed through the sight. Then, to ensure that the sight was centered on the rope and the star, I used "parallel sighting." I moved my head from side to side to make sure I could see Polaris emerge from behind the rope with either side movement of my head. When I could, I knew the sight was centered.

On the night of October 16, 2012, I waited for Polaris to move to its eastern elongation, performed the sighting and marked the location of the center of the sight on the bench with a small nail. On that date, Polaris would not reach its western elongation before dawn. I waited until November 4, 2012, when Polaris would reach its western elongation in darkness. I performed the second measurement, marking that location on the bench as well. I measured the distance between the two nails, divided that in two and placed a third nail at that point. The angle between the third nail and the base of the rope was my estimate of the meridian.

A total station can be used to locate the meridian to an accuracy of better than 20 seconds of arc. Using the total station to evaluate my results, I measured the horizontal angle between the base of the rope and each of the three nails. Nail 1, representing Polaris's eastern elongation, lay on a line 55.83 minutes east of due north. According to data from the US Naval Observatory, Polaris was at 55.01 minutes east of due

Below: Finding the meridian. Petrie's method (on the facing page) shown from above.



Below: Testing Petrie's method in Pomfret, CT (latitude N 41 degrees, 54 minutes, 7.3 seconds; longitude W 71 degrees, 58 minutes, 13.6 seconds; elevation 182 meters above mean sea level). A wooden pole set up on a second story porch substituted for Petrie's masonry wall. The pole held a plumb line that was fixed at its base. The observer sights a star through the vanes of the sliding sight. Photo by Joan Dash.



Glen Dash demonstrates during the day how he tested Petrie's method at night.

north when I measured it. My measurement error was 0.82 minutes. Nail 2, which recorded Polaris' western elongation, lay on a line 55.70 minutes west of due north. At the moment I measured it, Polaris, according to the Observatory, was 54.61 minutes west of due north for an error of -1.09 minutes. As it turned out, these errors nearly canceled. The horizontal angle formed by Nail 3 and the base of my rope, my predicted meridian, was in error by just -0.14 minutes of arc.

The fact that the errors from the two measurements nearly canceled out was probably fortuitous. Nonetheless, I had recorded both eastern and western elongations of Polaris to just over 1 minute of arc. The Egyptians, of course, would not have had the advantage of my modern ropes and would have had to have used Thuban, which was dimmer and farther away from the pole. Nonetheless, I believe they could have used Petrie's method to find due north to within 1 to 2 minutes of arc.

Nor would the Egyptians have needed a wall and plumb line 50 feet high. Petrie wanted to prove that the Egyptians could have located the meridian to within 15 seconds of arc—one quarter of one minute. However, as we shall see, the Great Pyramid is 5 to 6 minutes of arc from due north. They could have achieved that accuracy with a far shorter plumb line.

Spence's Transits

Kate Spence, of the University of Cambridge, thought that the Egyptians may have used a technique known as "simultaneous transit" (facing page) to orient the pyramids. She noted that two bright stars—Kochab in the Little Dipper and Mizar in the Big Dipper—straddled the celestial pole in the pyramid age.¹⁰ In fact, in 2467 BC a cord drawn through them would have

passed directly through the pole. An observer in 2467 BC could have held up a plumb line and waited for the two stars to transit behind it. At that moment, the line between the observer's pupil and the plumb line would have been the meridian.

For any two stars, however, this technique only works perfectly in one particular year. Owing to precession—a wobble in the Earth's orientation as it spins on its axis—the celestial pole moves relative to the stars. In the case of the simultaneous transit of Kochab and Mizar, this movement amounts to 31 minutes of arc per century or about 3 minutes of arc per decade.¹¹ Therefore, if the observer repeated the same measurement ten years later in 2457 BC, his or her results would have been off by about 3 minutes of arc.

To Spence, however, this was an advantage. The effect of precession on the movement of Kochab and Mizar relative to the celestial pole could be used to provide the very date the Great Pyramid was started. To calculate that date, Spence used Josef Dorner's measurement of the azimuth of the lower edge of the casing on the pyramid's west side.¹² Dorner, an Austrian surveyor and archaeologist, measured it as 2.8 minutes of arc west of north.* The two stars aligned 2.8 minutes of arc west of north in 2476 BC. According to Spence, that date, plus or minus five years, was the date the Great Pyramid was started.

We can use Spence's theory to calculate the commencement date of other 4th Dynasty pyramids as well. Egypt's 4th Dynasty (c. 2575–2465 BC) was the pinnacle of its pyramid-building age and included all three pyramids at Giza (Khufu, Khafre, and

*This measurement was likely in error by about 30 seconds of arc making it -3.3 minutes, as discussed in "New Angles on the Great Pyramid," *AERAGRAM* 13-2, Fall 2012, page 17. I have used the number Spence cited here.

Commencement Date of 4th Dynasty Pyramids Based on the Angle of the Casings and Calculated using Spence's Theory¹

PYRAMID	SNEFRU-MEIDUM	SNEFRU-BENT	SNEFRU-RED	KHUFU	KHAFRE	MENKAURE
Spence's Tabulated Azimuths of Casings (Minutes of Arc) ²	-18.1	-11.8 ⁵	-8.7 ⁶	-2.8	+6.0 ⁷	+14.1
Date of Commencement According to Spence's Theory ³	2525 BC	2505 BC	2495 BC	2476 BC	2448 BC	2422 BC
Date of Commencement According to Spence's "Currently Accepted" Chronologies ⁴	2598 BC	2583 BC	2572 BC	2552 BC	2520 BC	2487 BC
Difference in Years	73	78	77	76	72	65
Order of Commencement	1	2	3	4	5	6

1. The theory is described in Spence, K., "Ancient Egyptian chronology and the astronomical orientation of the pyramids," *Nature*, 408, 16 November 2000, pages 320–324.

2. Spence used the west side of each pyramid except for the Red Pyramid, where there was no data on the west side. She used the east side instead.

3. These dates of commencement differ from Spence's tabulated dates of accession by two years since she assumed (as I did here) that the Meidum and Giza pyramids were started in the second year of the king's reign.

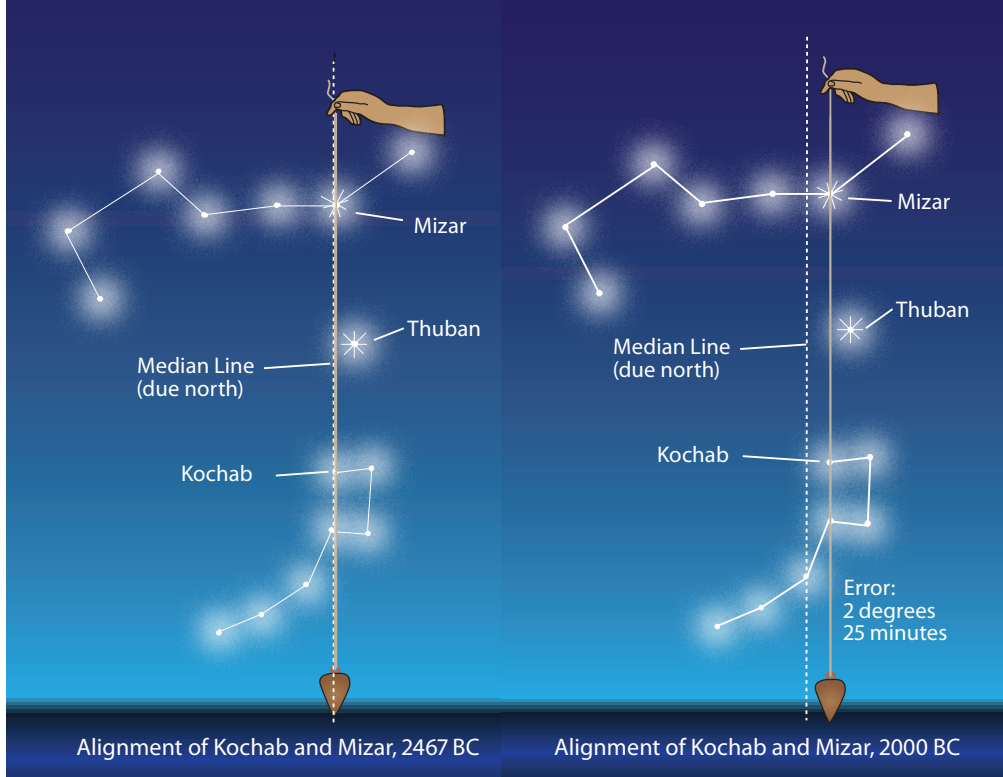
4. Spence derived her "currently accepted" chronology by using "von Beckerath's chronology (lower estimates) with the exception of the length of Snofru's reign and the dates of construction of his pyramids ... which follow Stadelmann" (page 320).

5. Spence cited Dorner for this number. He, however, says that due to settling,

the original lines of the Bent Pyramid could not be determined. Dorner was able to determine the original azimuth of the north side as -8.4 minutes off due east-west. If we assume the base of the pyramid was square, then the best estimate we have for the west side would be -8.4 minutes as well. Dorner, J., "Form und Ausmaße der Knickpyramide" *Mitteilungen des Deutschen Archäologischen Instituts*, Abteilung Kairo, 42, 1986, page 51.

6. Dorner, J., "Neue Messungen an der Roten Pyramide," in *Stationen: Beiträge zur Kulturgeschichte Ägyptens*; Rainer Stadelmann *Gewidmet*, ed. by H. Guksch and D. Polz, Mainz: Von Zabern, 1998, page 23.

7. The actual azimuth of the east side of the Khafre Pyramid is -6.0 minutes of arc according to Dorner. Spence reversed the sign based on her theory that the measurements for this pyramid were made when Kochab was in upper culmination and Mizar was in lower.



Spence's simultaneous transit. An observer holding up a plumb line in 2467 BC would find Mizar and Kochab aligned with the meridian. By 2000 BC, the alignment would have shifted east. Spence used this not only to explain pyramid alignments, but to predict the dates the pyramids were built.

Menkaure) and the three pyramids of Snefru, Khufu's father, the first at Meidum, and the second and third, the "Bent" and the "Red," at Dahshur. The table on the facing page compares calculated dates for the commencement of these six pyramids using Spence's methodology with more traditional dates based on the work of von Beckerath¹³ and Stadelmann.¹⁴ Spence cites their work in her paper as a "currently accepted" chronology.

Spence's analysis places the 4th Dynasty pyramids in their correct order of commencement. On average, Spence's dates are 74 years later than those of the currently accepted chronology.†

Spence defends the difference stating that, "[E]xisting Egyptian chronologies of this period [which are] based primarily

on cumulative reign lengths can only be considered accurate to about ±100 years."

Thus the casings may have been aligned with due north using simultaneous transit. However, by applying the same analysis to the descending passageways we can show that these passageways were aligned using some other method. The descending passageways of the Bent and Red Pyramids are aligned to due north with extraordinary precision, -0.5 and +2.9 minutes of arc respectively, even better than that of the Great Pyramid. As seen in the table below, applying the simultaneous transit analysis to the azimuths of the descending passageways produces an order of commencement that is wrong. The analysis would have the Great Pyramid construction starting before Snefru's last two pyramids. Such an order of commencement cannot be reconciled

with the historical record. Whatever method the Egyptians used to align the descending passageways of these pyramids with due north, it was not simultaneous transit.

In 1981 Josef Dorner said that the pyramid's extraordinary alignments could only have been achieved by measuring a circumpolar star at its elongations.¹² In 1998 he noted that the

†The gap of 74 years has troubled some. Juan Antonio Belmonte has proposed that the Egyptians might have used Megrez and Phekda in the Big Dipper instead of Kochab and Mizar. Using those stars moves the commencement date for the Great Pyramid to approximately 2550 BC. Belmonte, J. A., "On the Orientation of the Old Kingdom Egyptian Pyramids," *Archaeoastronomy* 26 (JHA, xxxii, 2001), pages s1-s20.

Commencement Date of 4th Dynasty Pyramids Based on Based on the Angle of the Descending Passageways and Calculated using Spence's Theory⁷

PYRAMID	SNEFRU-MEIDUM	SNEFRU-BENT	SNEFRU-RED	KHUFU	KHAFRE	MENKAURE
Azimuths of the Descending Passageways (Minutes of Arc)	-21.6' ²	-0.5' ³	+2.9' ⁴	-5.8' ⁵	+5.6' ⁶	+13.3' ⁷
Date of Commencement According to Spence's Theory	2537 BC	2469 BC	2458 BC	2485 BC	2449 BC	2424 BC
Date of Commencement According to Spence's "Currently Accepted" Chronologies	2598 BC	2583 BC	2572 BC	2552 BC	2520 BC	2487 BC
Difference in Years	61	114	114	67	71	63
Order of Commencement	1	3	4	2	5	6

1. The theory is described in Spence, K., "Ancient Egyptian chronology and the astronomical orientation of the pyramids," *Nature*, 408, 16 November 2000, pages 320-324.

2. Petrie, W. M. F., *Medum*, London: David Nutt, 1892, page 11.

3. Dorner, J., "Form und Ausmaße der Knickpyramide" *Mitteilungen des Deutschen Archäologischen Instituts, Abteilung Kairo*, 42, 1986, page 42.

4. Dorner 1998, page 27. See note 6, table on facing page.

5. Petrie, W. M. F., *The Pyramids and Temples of Gizeh*, London: Field and Tuer, 1883, page 125.

6. Petrie 1883, page 125. We have reversed the sign. As noted, Spence reversed the sign based on her theory that the measurements for this pyramid were made when Kochab was in upper culmination and Mizar was in lower.

7. Petrie 1883, page 117.

sloping surface of the descending passageways actually made that easier.¹⁵ Instead of needing a plumb line 10 to 15 meters long, the pyramid builders only needed to suspend a short plumb line from the top of the passageway to its base (as shown on the right). A shorter plumb line is more stable, producing more accurate measurements. To lay out the descending passageways in the Bent Pyramid, the builders would first have cut a trench into the desert floor roughly aligned north-south. They then could have used the circumpolar method to draw a meridian line down the center of the trench. Finally, they would have laid masonry walls into the trench parallel with the meridian line to finish the passageway. At Khufu, things were a bit more complicated since the entire pyramid is built over a bedrock knoll. Here, the Egyptians could have first built a rough masonry passageway over the bedrock knoll roughly aligned with due north. Then they could have taken the meridian and finished the masonry portion of the passageway with fine stone laid parallel with the meridian. To finish the lower portion of the passageway, they then would have had to bore into the bedrock along a line defined by the angle of the upper passageway. Dorner believed that the Egyptians could have aligned the casings using the same method, but with a longer and less stable plumb line. He speculated that the use of the longer plumb line could account for the greater error in the casing's azimuths.

Conclusions

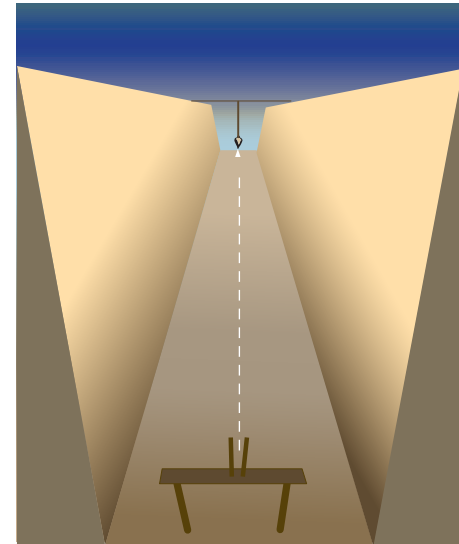
The Egyptians most likely used a circumpolar star to align the descending passageways of the Bent, Red, Great, and Khafre Pyramids with due north. Petrie believed the Egyptians used Thuban, the pole star of their time. Using Polaris, I have demonstrated that Petrie's technique is practical.

As for the casings, the Egyptians may or may not have used the same method. If they did, the longer plumb line required may explain why the casings are not aligned with due north as well as the descending passageways. On the other hand, Spence's data does seem to demonstrate a link between the orientations of the casings and the movement of stars in simultaneous transit. Finally, we cannot completely exclude the possibility the Egyptians aligned at least some of their pyramids with due north by using the sun, particularly those in error by 20 minutes of arc or more.

ACKNOWLEDGEMENTS

I would like to thank Dr. Joan Dash for her patient reading and re-reading of the text, her many suggestions, and her illustrations. I am also indebted to Dr. Juan Antonio Belmonte for his expeditious review of this paper and his comments. I also thank Dr. Peter der Manuelian of Harvard for hand-scanning and providing me a copy of Zbyněk Žába's important, but hard to find text.

Sighting the descending passageway. Petrie's method works best when sighting the descending passageway while it is being constructed. The plumb line required is short due to the passageway's upward angle of about 26 degrees, which lessens error. Once laid out, the passageway can be roofed, as was the case with the Bent and Red Pyramids, or extended into the bedrock, as was the case with the Great Pyramid.



REFERENCES

(Please note: references are numbered sequentially in the order in which they are cited in the text.)

1. Dash, G., "New Angles on the Great Pyramid," *AERAGRAM* 13-2, Fall 2012, pages 10-19.
2. Edwards, I. E. S., *The Pyramids of Egypt, rev. ed.*, London: Penguin Books, 1993.
3. Lehner, M., *The Complete Pyramids*, London: Thames & Hudson, 1997.
4. Allen, J., *The Ancient Egyptian Pyramid Texts*, ed. by Peter Der Manuelian, Atlanta: Society of Biblical Literature, 2005.
5. Shaltout, M., and J. A. Belmonte, "Introduction: Under Ancient Egyptian Skies," *In Search of Cosmic Order: Selected Essays on Egyptian Archaeoastronomy*, ed. by J. A. Belmonte and M. Shaltout, Cairo: Supreme Council of Antiquities Press, 2009, pages 13-26.
6. Isler, M., *Stick, Stones and Shadows: Building the Egyptian Pyramids*, Norman: University of Oklahoma Press, 2001.
7. Isler, M., "An Ancient Method of Finding and Extending Direction," *Journal of the American Research Center in Egypt* 26, 1989, pages 191-206.
8. Petrie, W. M. F., *The Pyramids and Temples of Gizeh*, London: Field and Tuer, 1883.
9. Žába, Z., *L'Orientation Astronomique dans L'Ancienne Égypte, et la Précession de l'Axe du Monde*, Prague: Czechoslovak Academy of Sciences, 1953.
10. Spence, K., "Ancient Egyptian chronology and the astronomical orientation of the pyramids," *Nature* 408, 16 November 2000, pages 320-324.
11. Rawlins, D., and K. Pickering, "Astronomical orientation of the pyramids," *Nature* 412, 16 August 2001, pages 699-700.
12. Dorner, J., "Die Absteckung und astronomische Orientierung ägyptischer Pyramiden," Ph.D. dissertation, Innsbruck University, 1981.
13. von Beckerath, J., *Chronologie des pharonischen Ägypten*, Mainz: von Zabern, 1997.
14. Stadelmann, R., *Die grossen Pyramiden von Giza*, Graz: Akademische Druck- und Verlagsanstalt, 1990.
15. Dorner, J., "Neue Messungen an der Roten Pyramide," *In Stationen: Beiträge zur Kulturgeschichte Ägyptens; Rainer Stadelmann Gewidmet*, ed. by H. Guksch and D. Polz, Mainz: Von Zabern, 1998, pages 23-30.